



Research article

Forecasting the Southeast Asian Currencies against the British Pound Sterling Using Probability Distributions

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ABSTRACT

The current study aimed to identify the most suitable probability distribution function (pdf) for modeling the exchange rates of three countries. Financial data is essential to many people and to the management of a country. Volatility in financial data influences individual and the country's economic growth. This volatility in the exchange rates between the Malaysian Ringgit (MYR), Singapore Dollar (SGD), and Thailand Thai Baht (THB) against British Pound Sterling (GBP) is found to be very high which make it difficult to model and forecast. This is what has necessitated the development of an accurate and reliable approach for assessing and reducing the risks of trading in any of these currencies.

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1. Introduction

The financial incidents in one part of the world can influence those on the other side since the world has become globalized. Trade occurs between people and between countries. The exchange rate between the two foreign currencies determines the value of one currency against the other. A competitive exchange rate has been one of the key drivers of economic growth in many countries. A country's foreign debt commerce, employment, balance of payments, and organization of production and consumption are all significantly affected by exchange rates [1]. Trading is carried

out in a common currency to promote trade efficiency as the various nations each have their currencies.

The GBP was recognized as the global trade currency in 1924, although the American dollar's market share was already substantial [2]. After the American dollar and the Euro, the Pound Sterling is now the third most popular currency in consideration for debt contracts worldwide [3].

According to [4], around 80% of outstanding foreign debt instruments in Southeast Asia's most important growing markets were denominated in foreign currencies in recent years. Additionally, most cross-border trade invoices from, to, and within Southeast Asia are issued in U.S. dollars as the main currency. For instance, between 2015 and 2020, most of the significant emerging market economies in Southeast Asia invoiced between 80 and 90% of their exports in foreign currencies [5]. The exchange rates between the GBP and the Southeast Asian countries have been very volatile. The effects of these fluctuations on these countries' economies are devastating. Researchers' motivation in foreign currency volatility can be related to the difficulties in forecasting future exchange rates using empirical methods [6]. Several researchers apply the Box-Jenkins approach for forecasting and modeling. Applying the Box-Jenkins technique for forecasting has its shortcomings. This approach relies on the assumption that the variables have a linear relationship. However, time series data are usually nonlinear in the real world [7]–[10]. Furthermore, the Box-Jenkins approach for model selection is a tedious method requiring computational skill and does not provide enough accuracy [11], [12]. As a result, the Box-Jenkins approach does not provide sufficient precision for modeling and forecasting.

Developed model-free forecasts of the correlation and volatility of the daily exchange rate using the exchange rate between the Yen and the Deutschmark against the American Dollar throughout the entire decade [13]. found that exchange rates have a longer tail than the normal distribution of the future price changes monitored at high frequency (daily/weekly) [14]. developed a new skewed probability model for Nigerian stock market volatility [15]. The model was the best to capture time-varying volatility in the stock market than other statistical models. developed a novel flexible model capable of capturing the asymmetries and long memory of the volatilities and accounting for the presence of dynamic skewness and kurtosis [16]. The applicability of the new model was tested using cryptocurrencies, and the forecasts were improved at various horizons.

In their study on modeling stock returns using the compound distribution for empirical comparison [17], observed that monthly returns fitted normal distribution appropriately. A study performed by found that the pattern of exchange rates is leptokurtic and compared to normal distribution [18]. An attempt was made to model several pdfs to the Standard & Poor's 500 (S&P 500) data, revealing that the 't' distribution provided a better fit [19]. It has long been assumed that financial stock returns are usually asymmetrical [20]. The distributions of daily stock returns are usually follow asymmetrical distributions [21], [22], regardless of how large the data is [21], [23]–[25].

In recent years, there have been notable advancements in the field of statistical analysis within environmental studies, particularly in the utilization of probability distribution models [26]. These statistical models have proven to be valuable decision support tools for assessing asset returns. The use of probability distribution models in statistical modelling of financial phenomena has a long history, and one of the latest advances in the statistical analysis of finance studies [27]. For instance, several researchers compared many probability distributions to find the one that best fit the empirical datasets. For instance, [28] used probability distributions to estimate the exchange rates between monthly Nigerian naira and British pound. Therefore, this study aims to identify the probability distribution function (pdf) that can adequately fit and forecast the exchange rates between the MYR, SGD, and THB against GBP. The study also investigated the practicability of symmetric and asymmetric probability distributions with financial application in exchange rate fitting and forecasting to tackle the difficulties in estimating financial data with better accuracy. Forecasting can be more effective, reliable, and accurate if a suitable pdf is identified. Instead, this research focused on the pdfs that the exchange rates follow. Over the last decade, determining the flexible probability distribution capable of forecasting different financial datasets has received a lot of attention in the field of exchange rates. Probability distribution functions are forward-looking since they are on actual data. Additionally, they do not require a significant historical data to be approximated efficiently. Moreover, pdfs are also excellent at capturing the underlying uncertainties in datasets [29]. Probability distribution functions are independent of mathematical priors [13].

The contributions of this paper include (i) to Identify an appropriate statistical forecasting model for Southeast Asian currencies' exchange rate against British Pound Sterling (ii) to demonstrate the flexibility of the identified model for forecasting the exchange rates of the three Southeast Asian selected currencies against the British Pound Sterling, and (iii) to carry out forecasts for each of the currencies' exchange rates using the identified probability model.

The rest of the paper proceed as follows: Section 2 provides the data source and a brief review of some commonly used probability distributions in finance. Section 3 illustrates the performance of the selected probability distributions for the three chosen currencies' exchange rates, simulation studies, and the in-sample forecasts for the three currencies' exchange rates. The discussions and conclusion are discussed in Sections 4 and 5, respectively.

2. Research Methods

The data on the monthly exchange rates between the MYR, SGD, and THB against GBP was assessed using several pdfs to determine which one best fits the data. The data was obtained from the Bank of England available at (<https://www.bankofengland.co.uk/boeapps/database/Rates>) from April 2005 to August 2022. The data from 2005 to 2020 was applied as the training set, and the data from 2021 to 2022 validate the model. The best model was selected and used for simulation and forecasting. The algorithm of the research is depicted in Figure 1.

2.1. Probability Distributions

In this section, the probability distributions used in this research study include the logistic, normal, gamma, log-normal, Weibull, log-logistic, exponential, Cauchy, inverse gamma, and log gamma distributions. Brief reviews of these distributions are discussed in the following paragraphs.

Logistic (L) distribution

A random variable X is said to follow a logistic distribution with the location parameter μ and the scale parameter s , if its pdf is given by:

$$f(x; \mu, s) = \frac{e^{-\left(\frac{x-\mu}{s}\right)}}{s\left(1+e^{-\left(\frac{x-\mu}{s}\right)}\right)^2}, x \in \mathbb{R}, \mu \in \mathbb{R}, s > 0 \quad (1)$$

The mean and variance of the logistic distribution are respectively given as:

$$E(X) = E(\mu + sY) = \mu + sE(Y) = \mu \left(\because Y = \frac{X-\mu}{s} \right) \quad \text{and} \quad Var(\mu + sY) = s^2 Var(Y) = \frac{s^2 \pi^2}{3}.$$

2.1.1. Normal (N) distribution

A random variable X is said to have a normal distribution with the mean parameter μ and variance parameter σ^2 if its pdf is given by the probability law:

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{x-\mu}{\sigma}\right)^2}; x \in \mathbb{R}, \mu \in \mathbb{R}, \sigma > 0. \quad (2)$$

The mean and variance of the normal distribution are respectively given as:

$$E(X) = \mu \text{ and } Var(X) = \sigma^2.$$

2.1.2. Gamma (G) distribution

A random variable X is a gamma distribution with the shape parameter α and the rate parameter β , if its pdf is given by:

$$f(x; \alpha, \beta) = \begin{cases} \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, & 0 < x < \infty, \alpha > 0, \beta > 0 \\ 0, & x < 0 \end{cases} \quad (3)$$

The mean and variance of the gamma distribution are respectively given as:

$$E(X) = \frac{\alpha}{\beta} \text{ and } Var(X) = \frac{\alpha}{\beta^2}.$$

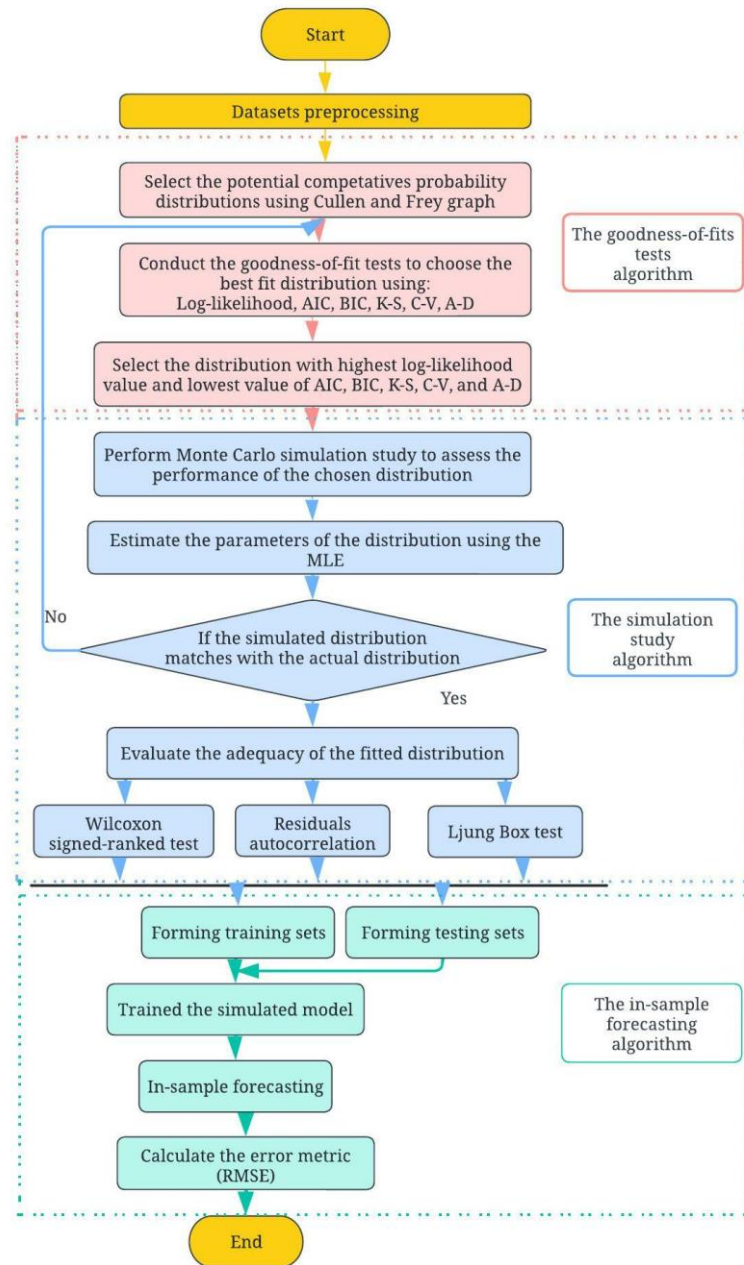


Figure 1. Framework of the study.

2.1.3. Log-normal (LN) distribution

The positive random variable X is said to have a log-normal distribution if $\log_e X$ is normally distributed, its pdf is given by:

$$f_x(y) = \begin{cases} \frac{1}{y\sigma\sqrt{2\pi}} \cdot e^{-\left(\frac{\log y - \mu}{\sigma}\right)^2}, & y > 0 \\ 0, & y \leq 0 \end{cases} \quad (4)$$

The mean and variance of the log-normal distribution are respectively given as:

$$E(X) = e^{\left(\mu + \frac{\sigma^2}{2}\right)} \text{ and } Var(X) = [e^{(\sigma^2)} - 1]e^{(2\mu + \sigma^2)}.$$

2.1.4. Weibull (W) distribution

A random variable X has a Weibull distribution with the scale parameter α and the shape parameter c if its pdf is:

$$f(x; \alpha, c) = \begin{cases} \frac{c}{\alpha} \left(\frac{x}{\alpha}\right)^{c-1} e^{-\left(\frac{x}{\alpha}\right)^c}, & x \geq 0, \alpha > 0, c > 0 \\ 0, & x < 0 \end{cases} \quad (5)$$

The mean and variance of the Weibull distribution are respectively given as:

$$E(X) = \alpha \Gamma\left(1 + \frac{1}{c}\right) \text{ and } Var(X) = \alpha^2 \left[\Gamma\left(1 + \frac{2}{c}\right) - \left(\Gamma\left(1 + \frac{1}{c}\right)\right)^2 \right].$$

2.1.5. Log-logistic (LL) distribution

A random variable X is said to follow log-logistic distribution with the scale parameter α and the shape parameter β if its pdf is given by:

$$f(x; \alpha, \beta) = \frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{x}{\alpha}\right)^{\beta-1}}{\left(1 + \left(\frac{x}{\alpha}\right)^\beta\right)^2}, 0 < x < \infty, \alpha > 0, \beta > 0 \quad (6)$$

The mean and variance of the log-logistic distribution are respectively given as:

Writing $b = \frac{\pi}{\beta}$ for convenience, the mean is

$$E(X) = \frac{\alpha b}{\sin b} \text{ and } Var(X) = \alpha^2 \left(\frac{2b}{\sin 2b} - \frac{b^2}{\sin^2 b} \right), \beta > 2.$$

2.1.6. Exponential (Ex) distribution

A random variable X is said to have an exponential distribution with the rate parameter λ if its pdf is given by:

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \lambda > 0 \\ 0, & x < 0 \end{cases} \quad (7)$$

The mean and variance of the exponential distribution are respectively given as:

$$E(X) = \frac{1}{\lambda} \text{ and } Var(X) = \frac{1}{\lambda^2}.$$

2.1.7. Cauchy (C) distribution

A random variable X is a Cauchy distribution with the location parameter x_0 and the scale parameter γ if its pdf is given by:

$$f(x; x_0, \gamma) = \frac{1}{\pi \gamma \left[1 + \left(\frac{x - x_0}{\gamma} \right)^2 \right]}, x \in \mathbb{R}, x_0 \in \mathbb{R}, \gamma > 0. \quad (8)$$

This distribution does not have a defined mean and defined variance.

2.1.8. Inverse Gamma (IG) distribution

A continuous random variable X is said to have an inverse gamma distribution with the shape parameter α and the scale parameter β if its pdf is of the form:

$$f(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\left(\frac{\beta}{x}\right)}, 0 < x < \infty, \alpha > 0, \beta > 0 \quad (9)$$

The mean and variance of the inverse gamma distribution are respectively given by:

$$E(X) = \frac{\beta}{\alpha-1}, \alpha > 1 \text{ and } Var(X) = \frac{\beta^2}{(\alpha-1)^2(\alpha-2)}, \alpha > 2.$$

2.1.9. Log-gamma (LG) distribution

A random variable X is said to have a log-gamma distribution if its pdf is given as:

$$f(x; \alpha, \beta, \lambda) = \frac{(\log[x - \lambda + 1])^{\alpha-1} (x - \lambda + 1)^{-\left(\frac{1+\beta}{\beta}\right)}}{\beta^\alpha \Gamma(\alpha)}, 0 < x < \infty, x \geq \lambda, \alpha > 0, \beta > 0 \quad (10)$$

The mean and variance of the log-gamma distribution are respectively given by:

$$E(X) = (1 - \beta)^{-\alpha} + \lambda - 1, \beta < 1 \text{ and } Var(X) = (1 - 2\beta)^{-\alpha} - (1 - \beta)^{-2\alpha}, \beta < \frac{1}{2}.$$

2.2. Evaluating goodness-of-fit

In this section, we considered three commonly used statistical tests (analytical measures) to determine which statistical model best fits the exchange rates data. The measures are computed to compare the models. A statistical distribution with the highest negative log-likelihood value and the smallest Akaike information criteria (AIC) and Bayesian information criteria (BIC) values shows a superior model compared to other fitted statistical models [30]–[32].

3. Results and Discussion

3.1 Descriptive statistics

The data was the monthly exchange rates between the MYR, SGD, and THB against GBP from April 2005 to August 2022. The data was divided into the training and the testing sets.

Table 1 provides a descriptive statistic of the exchange rate data. GBP/MYR ranged from 4.62 to 7.254, with an average of 5.63 and a standard deviation of 0.65, GBP/SGD from 1.66 to 3.13, with an average of 2.14 and a standard deviation of 0.43, and GBP/THB from 37.39 to 74.41, with an average of 51.24 and a standard deviation and 9.48. The result shows that the MYR, SGD, and THB depreciated against the GBP over the research period. The high standard deviation obtained suggested the presence of volatility in the data. As observed from Table 1, the data sets are all positive skewed with negative kurtosis, which indicates non-normality patterns.

Table 1. Descriptive statistics of GBP/MYR, GBP/SGD, and GBP/THB exchange rates.

Currency	Min.	Q1	Q2	Mean	Q3	Max.	S.D.	Skewness	Kurtosis
GBP/MYR	4.62	5.24	5.45	5.63	5.83	7.254	0.65	0.79	-0.43
GBP/SGD	1.66	1.80	2.01	2.14	2.25	3.13	0.43	1.19	-0.18
GBP/THB	37.39	43.83	49.28	51.24	54.81	74.41	9.48	0.91	-0.15

3.2. Fitting probability distributions to GBP/MYR exchange rate

In this section, we use the GBP/MYR exchange rate data to find a distribution that can provide the best model for the data. Table 1 shows some descriptive statistics relating to the observations, indicating that the data set is positively skewed and fat-tailed towards the right. The empirical density and cumulative distributions (cdf) in Figure 2 confirmed that the data is positively skewed.

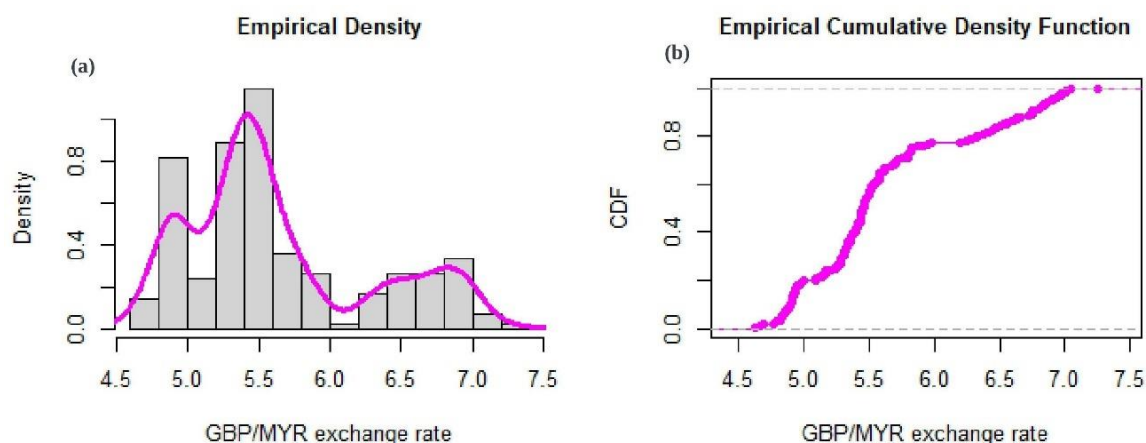


Figure 2. (a) Empirical density of GBP/MYR exchange rate (b) The cdf of GBP/MYR exchange rate.

Figure 3 shows the Cullen and Frey graph obtained using the bootstrapping method, identifying potential distributions that fit the data. However, the competing probability distributions used in this research include logistic, normal, gamma, log-normal, Weibull, log-logistics, exponential, Cauchy, inverse gamma, and log-gamma distributions.

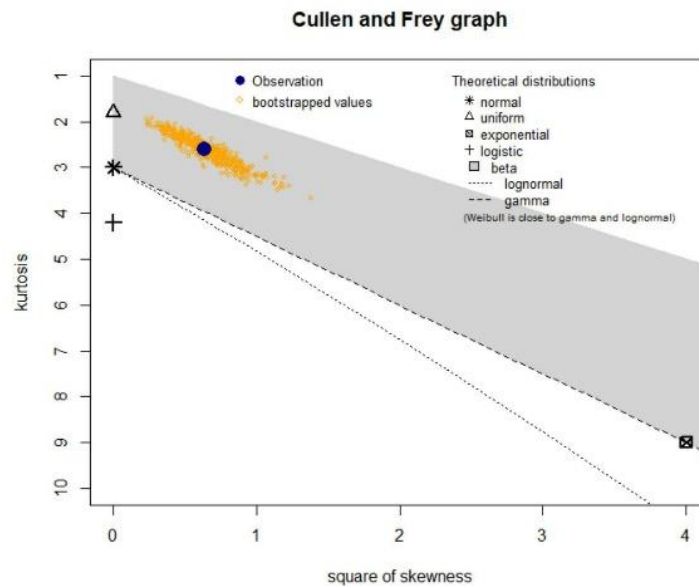


Figure 3. Graph of the skewness and kurtosis coefficients for the GBP/MYR exchange rate.

Table 2 gives the estimated values with standard errors (SE) in parentheses for all competing distributions using the maximum likelihood (ML) method. The negative log-likelihood (L), AIC, and BIC are computed to compare the performance of the models. The results revealed that the log-logistic has the highest value of negative log-likelihood and the lowest values of AIC and BIC, indicating the better fit distribution. This indicated that the exchange rate between the MYR and GBP fits the Log-logistic model than the other models.

Table 2. Negative log-likelihood values (L), MLEs of model parameters, the corresponding SEs (in parentheses), AIC and BIC statistics of the monthly MYR to GBP exchange rate.

Model	Estimates			Statistic		
	Location	Shape	Scale/Rate	L	AIC	BIC
Logistic	5.550 (0.044)	0.365 (0.021)	—	-207.253	418.506	425.190
Normal	5.629 (0.045)	0.646 (0.032)	—	-205.333	414.666	421.350
Gamma	—	79.621 (7.772)	14.146 (1.385)	-199.376	402.751	409.436
Log-Normal	1.722 (0.008)	0.111 (0.005)	—	-196.831	397.663	404.347
Weibull	—	8.562 (0.431)	5.930 (0.051)	-224.756	453.512	460.197
Log-Logistic	—	22.741 (0.917)	5.533 (0.042)	-193.998	391.397	398.081
Exponential	—	—	0.178 (0.012)	-570.122	1142.244	1145.586
Cauchy	5.420 (0.027)	—	0.282 (0.029)	-220.443	444.885	451.570
Inverse Gamma	—	83.376 (8.138)	463.555 (45.387)	-194.537	393.074	399.759
Log-Gamma	—	246.405 (24.085)	143.127 (14.004)	-194.140	392.280	398.964

Table 3 presents the values of several test statistics used to evaluate the goodness of fit for the distributions. The lowest values of the goodness of fit statistics such as the Kolmogorov-Smirnov (K-S), Cramer-von Mises (C-V) and Anderson-Darling (A-D) for log-logistics distribution demonstrate that the log-logistic model fits the exchange rate data best among the competing distributions. These tests can be found in [33], [34] and [33]. Table 3 proved

that the log-logistic distribution provides a superior fit than the competing distributions. This shows that the exchange rate between the MYR and GBP can describe adequately by using the log-logistic distribution.

Table 3. The goodness of fit statistic of the monthly exchange rate, MYR into GBP.

Model	L	N	G	LN	W	LL	Ex	C	IG	LG
K-S	0.154	0.174	0.160	0.152	0.199	0.131	0.560	0.220	0.145	0.144
C-V	0.985	1.456	1.205	1.087	2.042	0.695	16.580	0.991	0.981	0.965
A-D	6.827	8.161	6.872	6.276	11.004	5.378	76.576	6.233	5.731	5.645

Figure 4 shows empirical and theoretical pdf, Q-Q, CDF, and P-P plots. These plots illustrate which model the data tends to fit. We can observe that the exponential distribution diverges from the empirical line, while the Cauchy model overfits the data. Hence, the closer empirical and theoretical lines in Fig. 4 demonstrate that the log-logistics distribution better fits the above data.

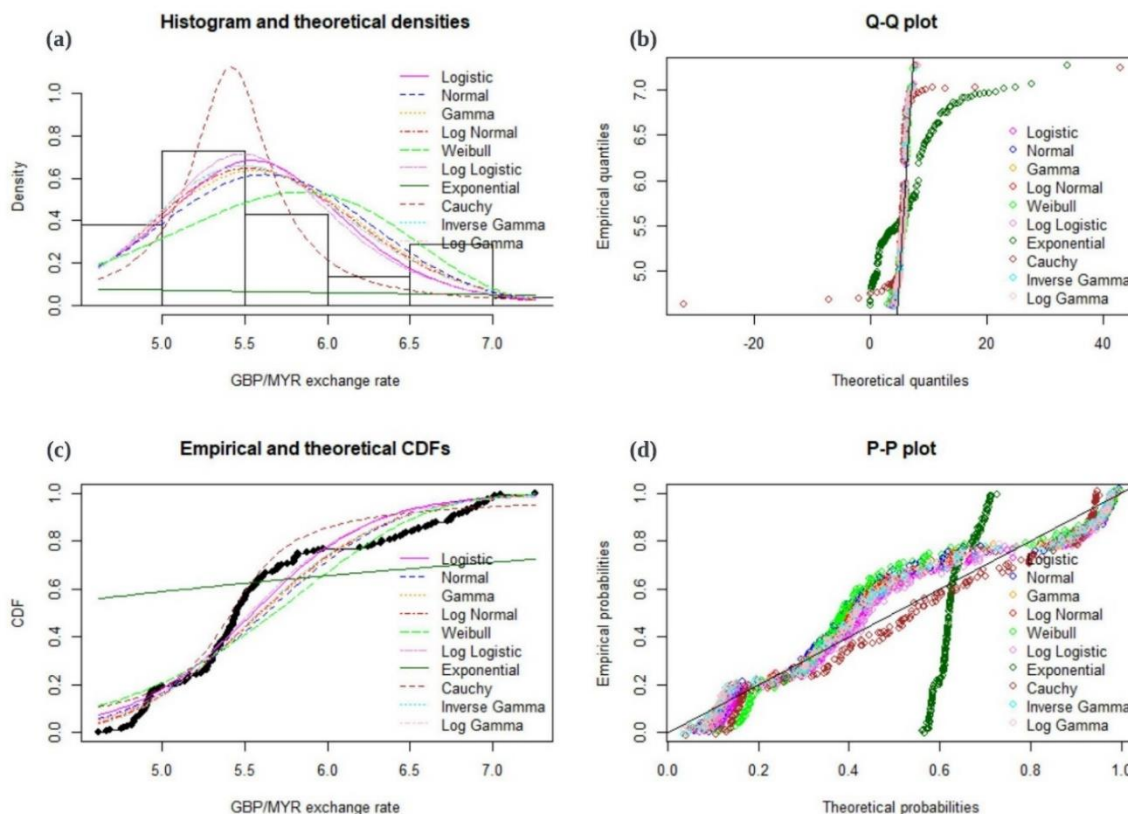


Figure 4. (a) The fitted pdfs on the histogram of GBP/MYR exchange rate along with their CDF (c), Q-Q (b), and probability plots (d).

Randomness and uncertainty are the fundamental components of probability. The simulation generates random values by replicating the reality of a random experiment by employing a suitable model developed based on a random experiment. A probability distribution used to describe an actual process that generates values of a certain quantity of interest can serve as a simple form of such a simulation.

Here, we perform Monte Carlo simulation tests to evaluate the performance of the fitted log-logistic distribution based on maximum likelihood estimates using the R programming language. To verify that the exchange rate data between the MYR and GBP follows the log-logistic distribution, the exchange rate data was generated from the simulated log-logistic distribution. The results from the simulation reveal that the simulated log-logistic distribution has a scale parameter value of 23.592 and a shape parameter value of 5.535. Data of size 209 has been generated from the simulated log-logistics distribution and compared with the real dataset. Figure 5 shows that the two datasets are comparable with each other. Therefore, the Log-logistic model is the most suitable model to fit this dataset.

The Wilcoxon signed-rank test with continuity correction was applied to verify whether the two dependent datasets were drawn from populations with the same distribution. The result from this test reveals that the two datasets are identical since the p-value is 0.4689, at a 5% significance level. Hence, we conclude that the exchange rate between the MYR and GBP follows the log-logistic distribution.

One method of accepting a developed pdf is to evaluate its empirical performance and ability to forecast accurately [13]. Therefore, to confirm the practicability of applying the log-logistics model for forecasting, the model's accuracy was tested using real and simulated data. For this purpose, the exchange rate between the MYR and GBP from 2016 to 2017 was forecasted using the log-logistic distribution. The training and the forecast data were compared (2021–2022). Two sample t-test was applied for the comparison. The research yielded a t-value of 1.1776 and a p-value of 0.25 at a 5% significance level. As a result, the datasets from the two models are identical.

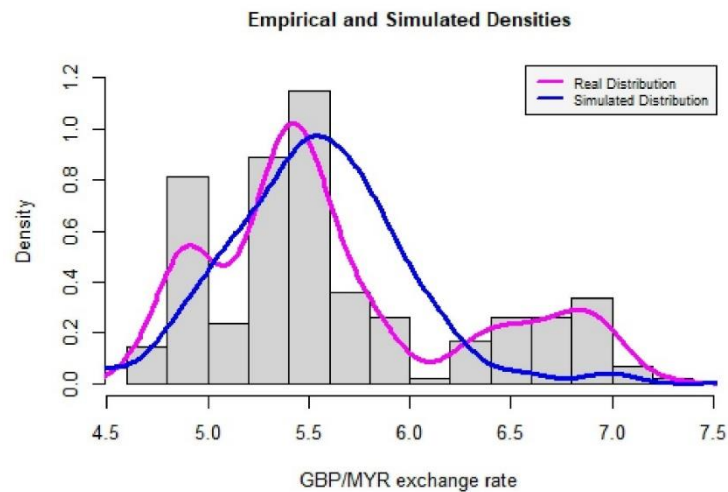


Figure 5. Graph of the distribution of original and simulated GBP/MYR exchange rate data.

Furthermore, the adequacy of the fitted model of the original and simulated probability distributions was verified via the autocorrelation of the residuals, as shown in Figures. 6 and 7, respectively. These plots illustrate the suitability of the actual and simulated fitted probability distributions.

Finally, we perform a statistical check using Ljung Box test to assess the autocorrelations of the residuals for the original distribution and the simulated distributions. The null hypothesis of this test assumed that the model does not show a lack of fit. In the Ljung Box test, if the p-value is less than 5% or 0.05 level of significance for the residuals, then we reject the null hypothesis and conclude that the model does show a lack of fit. The results obtained for p-values for both the original and simulated datasets are 0.9881 and 0.5225, respectively. Therefore, since the p-values of the residuals for the original and simulated datasets are greater than 0.05, we fail to reject the null hypothesis and conclude that the models are fine.

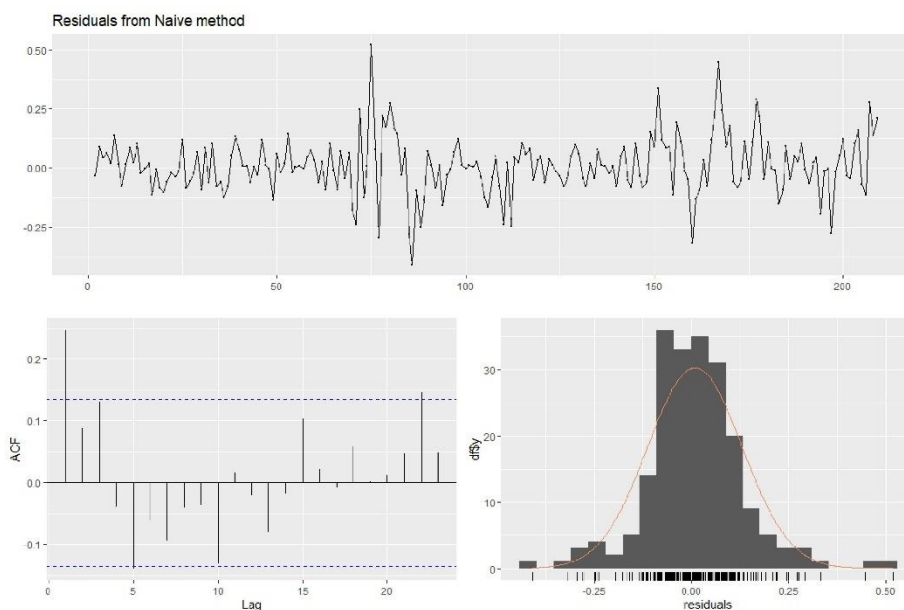


Figure 6. The residuals autocorrelation for the original distribution GBP/MYR exchange rate.

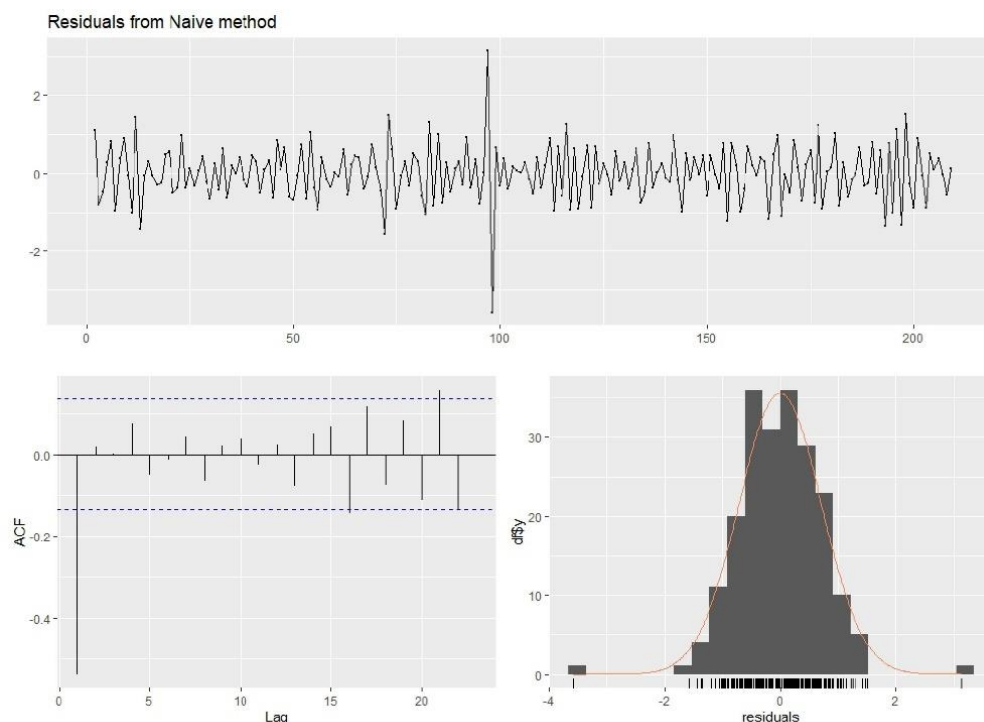


Figure 7. The residuals autocorrelation for the simulated distribution GBP/MYR exchange rate.

3.3. Fitting Probability distributions to GBP/SGD exchange rate

In this section, we use the SGD into GBP exchange rate data to identify a distribution that better fits the dataset.

The descriptive statistics in Table 1 shows that the data are significantly spread, positively skewed and platykurtic. The empirical density and cumulative distributions in Fig. 8 confirmed that the data is positively skewed.

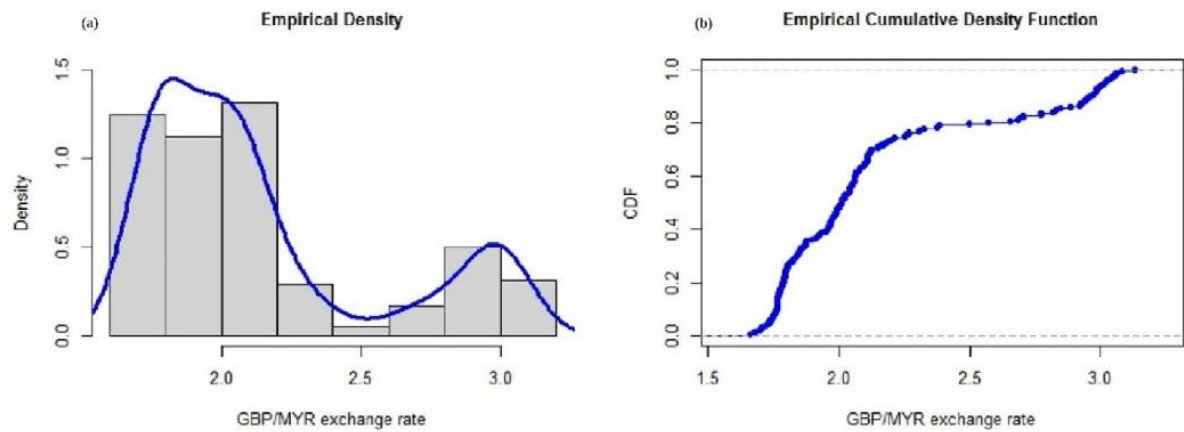


Figure 8. (a) Empirical density of GBP/SGD exchange rate (b) The cdf of GBP/SGD exchange rate.

Table 4 shows the competing distribution results by showing that the log-logistics distribution is the best distribution to describe the exchange rate between the SGD and GBP.

Table 4. Negative log-likelihood values (L), MLEs of model parameters, the corresponding SEs (in parentheses), AIC and BIC statistics of the monthly SGD to GBP exchange rate.

Model	Estimates			Statistic		
	Location	Shape	Scale/Rate	L	AIC	BIC
Logistic	2.069 (0.028)	0.232 (0.014)	—	-116.324	236.649	243.333
Normal	2.142 (0.029)	0.425 (0.021)	—	-117.857	239.713	246.398
Gamma	—	28.416 (2.764)	13.267 (1.302)	-103.514	211.029	217.713
Log-Normal	0.744 (0.013)	0.183 (0.009)	—	-97.347	198.694	205.379
Weibull	—	4.974 (0.249)	2.323 (0.034)	-131.981	267.961	274.646
Log-Logistic	—	11.718 (0.573)	2.050 (0.025)	-80.796	167.592	174.277
Exponential	—	—	0.467 (0.032)	-368.193	738.385	741.727
Cauchy	1.958 (0.019)	0.175 (0.016)	—	-114.365	232.730	239.415
Inverse Gamma	—	31.765 (3.091)	65.793 (6.453)	-91.757	187.514	194.199
Log-Gamma	—	18.289 (1.773)	24.581 (2.416)	-82.673	169.345	176.030

The Cullen and Frey graph in Figure 9 shows the potential positively skewed probability distributions that are more likely to fit the data. However, only the logistic, normal, gamma, log-normal, Weibull, log-logistics, exponential, Cauchy, inverse gamma, and log-gamma distributions will be examined to check whether the data has any of these competing distributions.

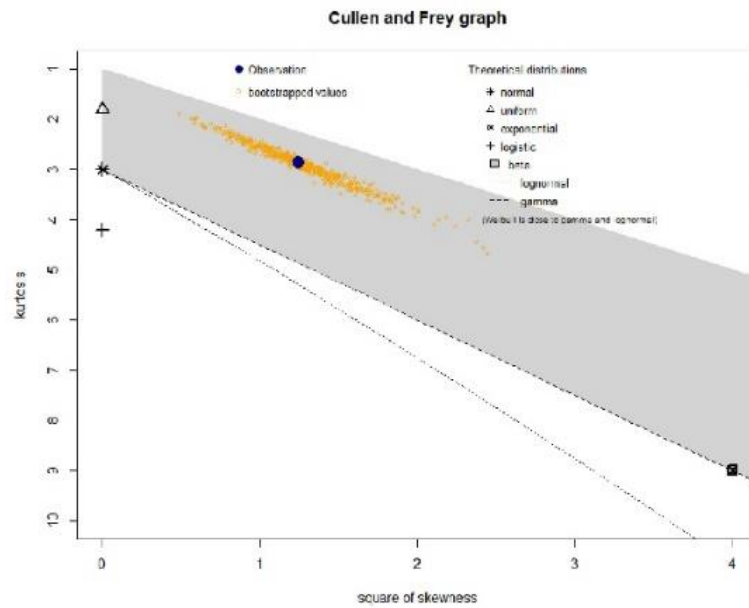


Figure 9. Graph of the skewness and kurtosis coefficients for the GBP/SGD exchange rate.

Table 5 prove that the log-logistics model best fits the exchange rate data among the competing distributions. Figure 10 shows a graphical performance of the fitted models by displaying pdf, Q-Q, CDF, and P-P plots. It can be observed in the empirical and theoretical lines indicate that the log-logistics distribution better fits the exchange rate between the SGD and GBP.

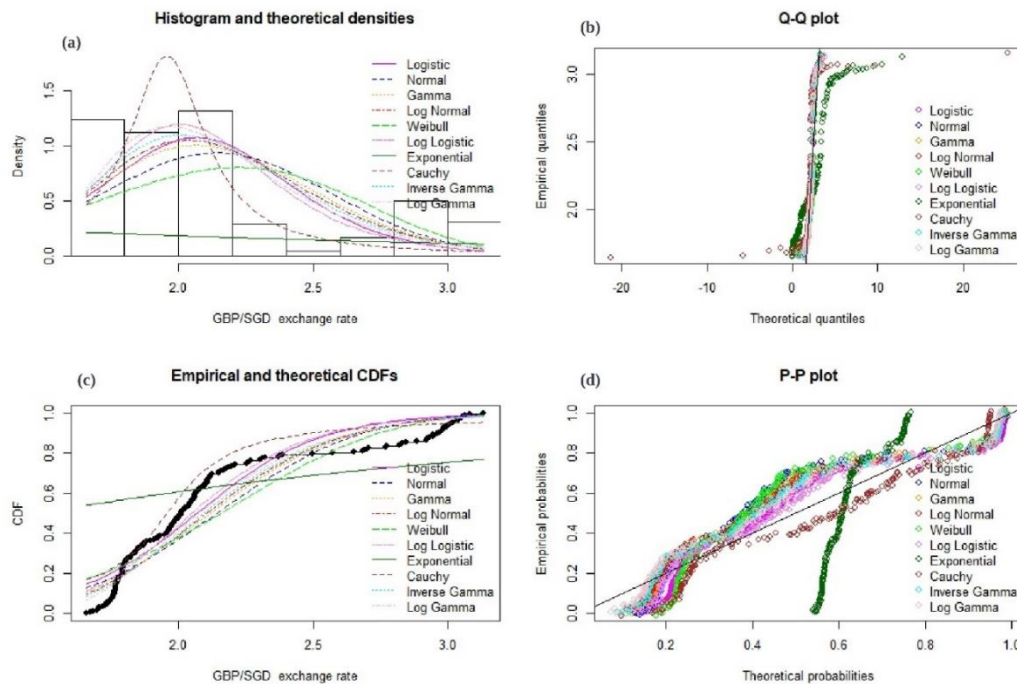


Figure 10. (a) The fitted pdfs on the histogram of GBP/SGD exchange rate along with their CDF (c), Q-Q (b), and probability plots (d).

Table 5. Goodness of fit statistic of monthly exchange rate, SGD into GBP.

Model	L	N	G	LN	W	LL	Ex	C	IG	LG
K-S	0.153	0.215	0.192	0.179	0.226	0.124	0.539	0.174	0.167	0.147
C-V	1.440	2.581	2.045	1.787	2.894	0.989	14.134	1.500	1.564	1.234
A-D	12.491	15.320	12.639	11.353	16.284	8.229	65.875	10.908	10.189	8.409

To verify that the exchange rate data between the SGD and GBP follows the log-logistic distribution, the exchange rate data was generated from simulated log-logistic distribution. The results from the simulation reveal that the simulated log-logistic distribution has a scale parameter value of 12.212 and a shape parameter value of 2.323. Data of size 209 has been generated from the simulated log-logistic distribution to compare with the real data (exchange rate between the SGD and GBP). Figure 11 depicts that the two datasets are comparable with each other. Therefore, the Log-logistic model best fit for this exchange rate data.

The Wilcoxon signed-rank test results reveal that the two datasets are identical since the p-value is 0.6868, at a 5% significance level. Hence, we conclude that the exchange rate between the SGD and GBP follows the log-logistics distribution.

The training and the forecast data were compared (2021–2022). Two sample t-test was applied for the comparison. The research yielded a t-value of 1.72 and a p-value of 0.13 at a 5% significance level. As a result, the datasets from the two distributions are identical.

The autocorrelation of the residuals was used to check the model's adequacy, as shown in Figures 12 and 13, respectively. These plots illustrate the suitability of the actual and simulated fitted probability distributions. The Ljung Box's result obtained for p-values for both the original and simulated datasets is 0.1723 and 0.32421, respectively. Therefore, since the p-values of the residuals for the original and simulated datasets are greater than 0.05, we fail to reject the null hypothesis and conclude that the models are fine.

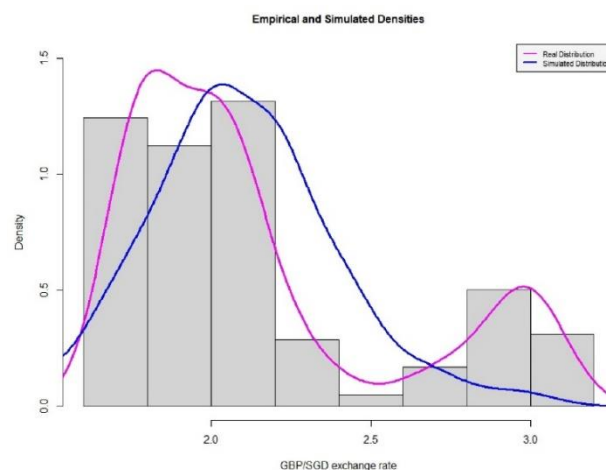


Figure 11. Graph of the distribution of original and simulated GBP/SGD exchange rate data.

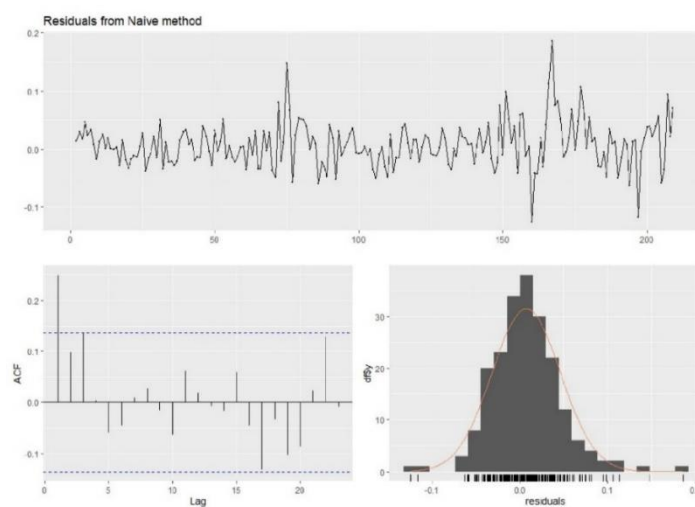


Figure 12. The residuals autocorrelation for the original distribution of GBP/SGD exchange rate.

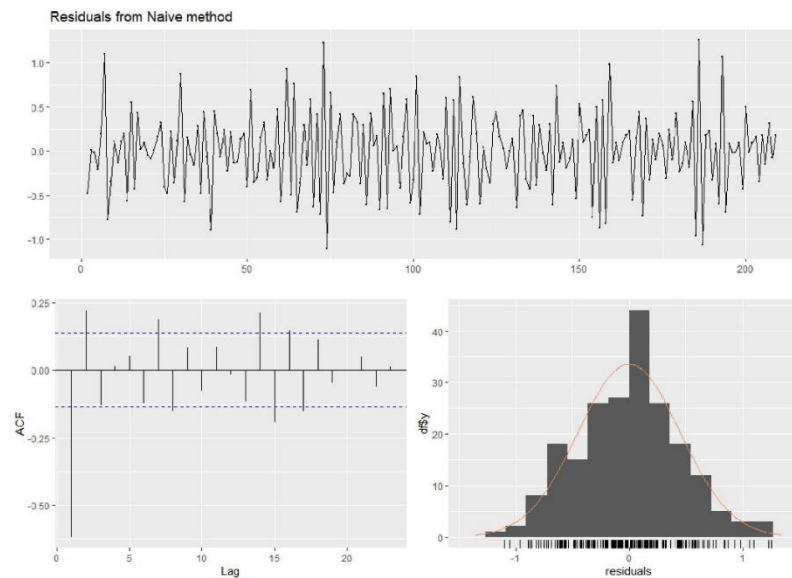


Figure 13. The residuals autocorrelation for the simulated distribution of GBP/SGD exchange rate.

3.4. Fitting probability distributions to GBP/THB exchange rate

In this section, we use the THB into GBP exchange rate data to find a distribution that can best model the data. Table 1 gives certain descriptive statistics of data under consideration revealing that the data are reasonably skewed and right tailed. Figure 14 confirms that the under-study data is positively skewed. Figure 15 reveals certain potential competitive probability distributions that are likely to fit the exchange rate data.

Table 6 displays parameter estimates. The higher value of the negative log-likelihood (L) and lower values of AIC and BIC indicate a better model. In this regard, it is evident from the result that the log-logistic model with the shape parameter value of 9.959 and the scale parameter value of 49.662 provides a better fit among the competitive models for the exchange rate between the THB and the GBP.

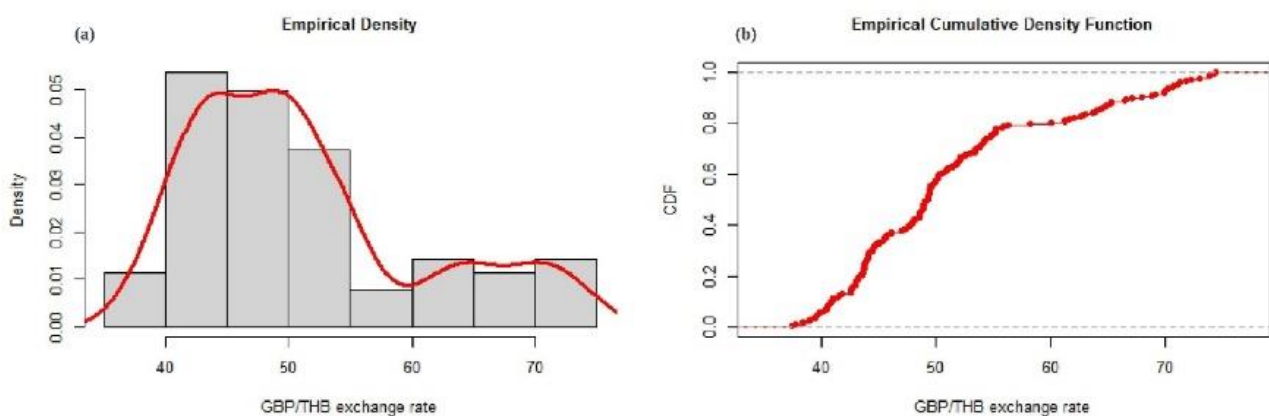


Figure 14. (a) Empirical density of GBP/THB exchange rate (b) The cdf of GBP/THB exchange rate.

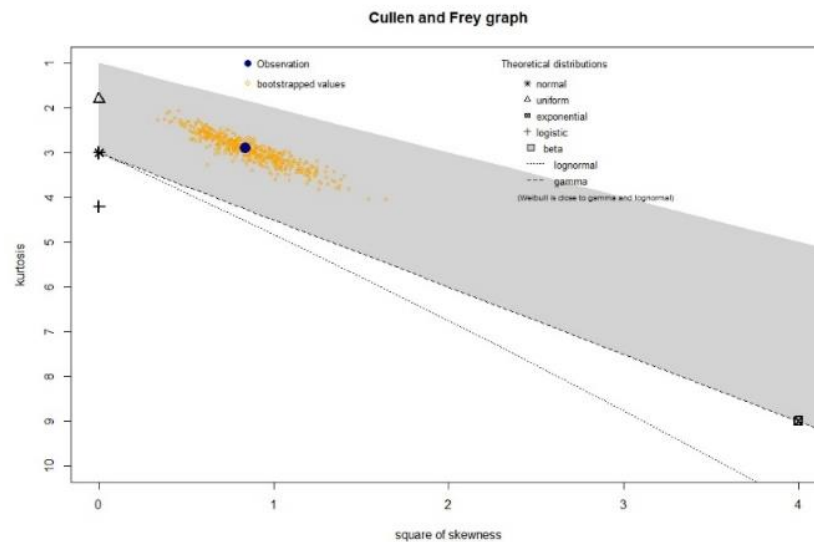


Figure 15. Graph of the skewness and kurtosis coefficients for the GBP/THB exchange rate.

Table 6. Negative log-likelihood values (L), MLEs of model parameters, the corresponding SEs (in parentheses), AIC and BIC statistics of the monthly THB to GBP exchange rate.

Model	Estimates			Statistic		
	Location	Shape	Scale/Rate	L	AIC	BIC
Logistic	50.059 (0.635)	5.281 (0.310)	—	-766.259	1536.518	1543.203
Normal	51.244 (0.654)	9.454 (0.462)	—	-766.071	1536.142	1542.827
Gamma	—	31.798 (3.094)	0.620 (0.061)	-755.576	1515.151	1521.836
Log-Normal	3.921 (0.012)	0.175 (0.009)	—	-751.452	1506.904	1513.589
Weibull	—	5.399 (0.270)	55.313 (0.754)	-780.025	1564.049	1570.734
Log- Logistic	—	9.959 (0.575)	49.662 (0.603)	-744.777	1483.554	1504.238
Exponential	—	—	0.019 (0.001)	-1031.75	2065.501	2068.844
Cauchy	48.240 (0.522)	4.818 (0.433)	—	-785.984	1575.967	1582.652
Inverse Gamma	—	34.173 (3.328)	1698.539 (166.602)	-747.986	1499.972	1506.657
Log-Gamma	—	511.759 (50.0541)	130.525 (12.773)	-749.580	1503.16	1509.845

Table 7 provides the values of goodness of fit test statistics. One can observed that the log-logistic distribution has the lowest values which suggest that this distribution fits the data best among the rest.

Table 7. Goodness of fit statistic of monthly exchange rate, THB into GBP.

Model	L	N	G	LN	W	LL	Ex	C	IG	LG
K-S	0.092	0.140	0.118	0.106	0.151	0.086	0.518	0.135	0.095	0.100
C-V	0.584	1.171	0.802	0.645	1.563	0.359	14.162	0.809	0.515	0.575
A-D	5.502	7.148	5.104	4.225	9.103	3.448	66.157	6.242	3.471	3.824

It can be observed that from empirical and theoretical pdf, Q-Q, CDF, and P-P plots given in Figure 16 that exchange rate between the THB and the GBP.

3.3.1. Simulation study

The exchange rate data between the THB and the GBP was generated using a simulated log-logistic distribution to verify that the data followed the underlying distribution. The results show that the log-logistics distribution has a scale parameter value of 10.273 and a shape parameter value of 49.974.

From Figure 17, it is evident that data generated from the simulated log-logistic model are closer to the original data suggesting that the log-logistic distribution can be used to approximate the under study.

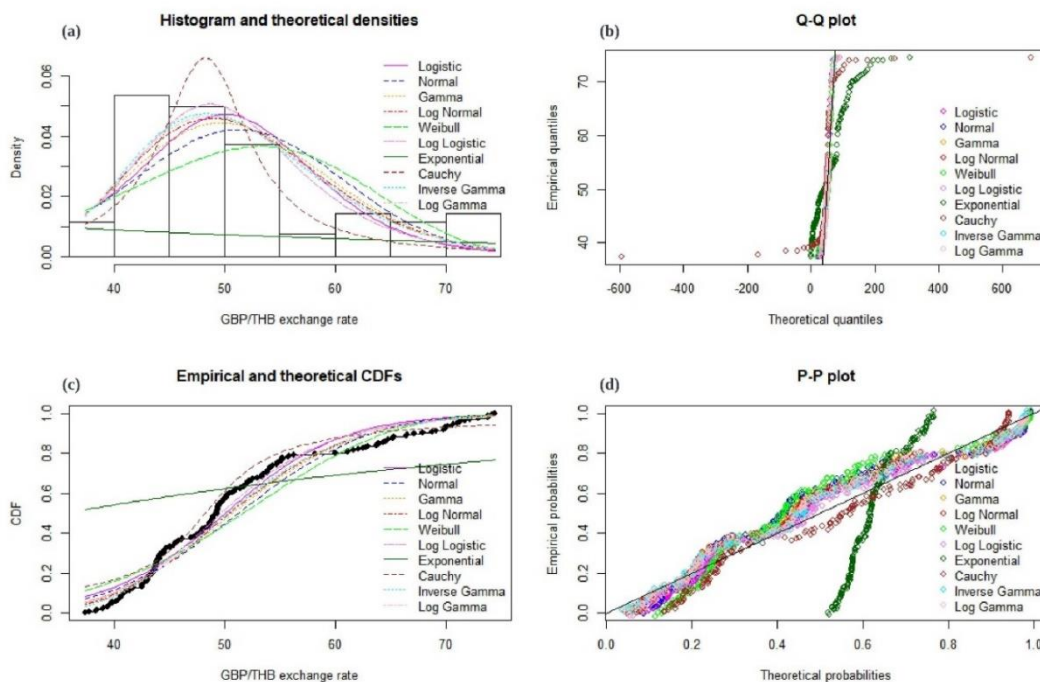


Figure 16. (a) The fitted pdfs on the histogram of GBP/THB exchange rate along with their CDF (c), Q-Q (b), and probability plots (d).

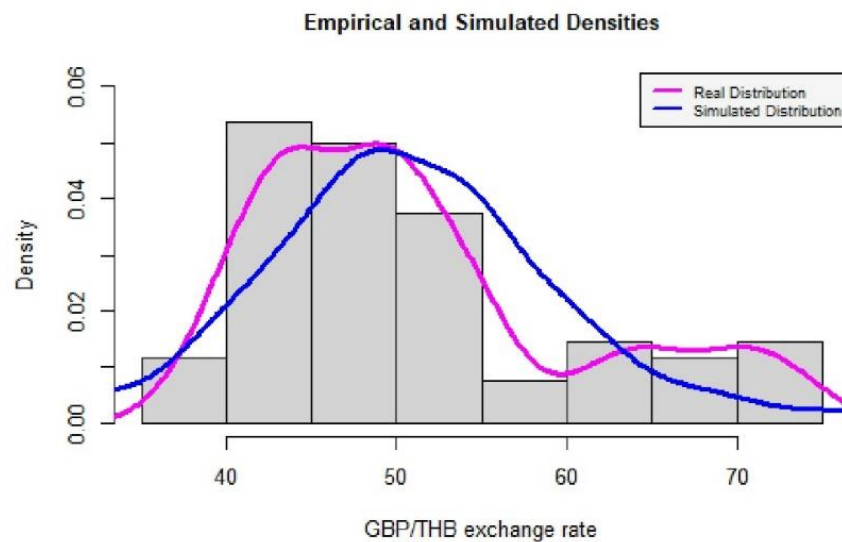


Figure 17. Graph of the distribution of original and simulated GBP/THB exchange rate data.

The Wilcoxon signed-rank test results reveal that the two datasets are identical since the p-value is 0.9645, at a 5% significance level. Hence, we conclude that the exchange rate between the THB and the GBP follows the log-logistics distribution.

The training and the forecast data were compared (2021–2022). Two sample t-test was applied for the comparison. The results revealed a t-value of -1.0534 and a p-value of 0.3054 at a 5% significance level. Thus, the datasets from the two distributions have the same distribution.

The adequacy of the fitted model of the original and simulated probability distributions was verified via the autocorrelation of the residuals, as shown in Figure 18 and 19, respectively. These plots illustrate the suitability of the actual and simulated fitted probability distributions.

A statistical check using Ljung Box test to assess the autocorrelations of the residuals for the original probability and the simulated distribution was performed. The results obtained suggested that the logistic model was adequate to fit the data.

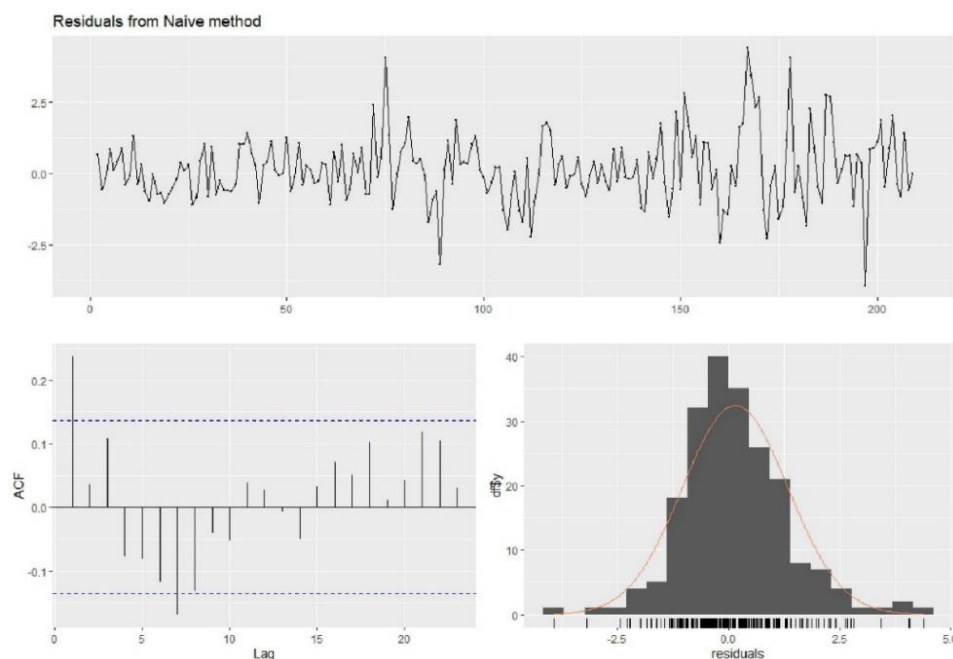


Figure 18. The residuals autocorrelation for the original distribution of GBP/THB exchange rate.

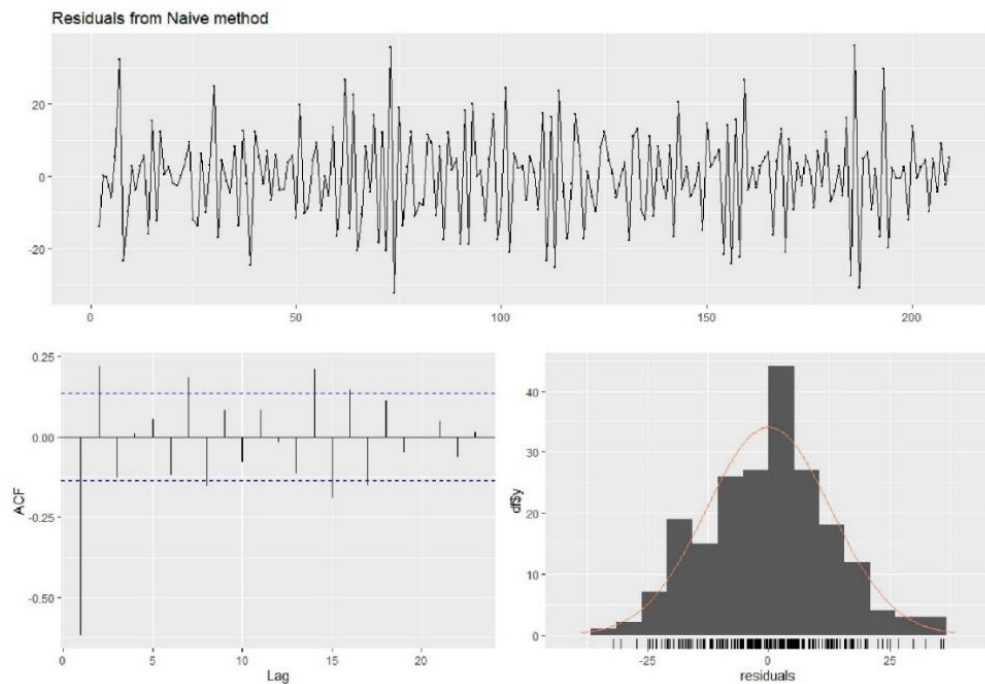


Figure 19. The residuals autocorrelation for the simulated distribution of GBP/THB exchange rate.

3.5. In-Sample Forecasting

In this section, we assess the ability of the fitted log-logistic model by performing a within-sample forecast. We use the testing dataset of the actual exchange rate data between the GBP/MYR, GBP/SGD, and GBP/THB from January 2021 to August 2022 and compare them to the corresponding forecast values of the simulated log-logistic model within the same period.

Table 8 shows the monthly values of the actual exchange rate data between the MYR and GBP with the forecasted values using the simulated log-logistic model. The root means square error (RMSE) was 1.66 for this dataset. The results of the forecast values were slightly close to the actual values, suggesting a remarkable performance of the chosen model for exchange rate forecasting.

Table 8. The in-sample forecast for the monthly values of the actual exchange rate data between the MYR and GBP with the forecasted values from Jan 2021 to August 2022.

Period (2021)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Original	5.32	5.36	5.41	5.45	5.51	5.52	5.53	5.61	5.61	5.62	5.67	5.68
Forecast	5.78	5.57	5.71	5.41	6.54	6.08	5.4	5.63	4.73	5.66	5.37	5.16
RMSE	0.11	0.05	0.07	0.01	0.23	0.13	0.03	0.01	0.2	0.01	0.07	0.12
Period (2022)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Original	5.78	5.76	5.67	5.72	5.83	5.84	5.82	5.83	---	---	---	---
Forecast	5.25	5.58	5.56	5.25	5.57	5.62	5.61	5.22	---	---	---	---
RMSE	0.10	0.03	0.03	0.11	0.05	0.04	0.16	0.14	---	---	---	---

Table 9 compares the monthly exchange rates between the SGD and the GBP to forecasted values based on a simulated log-logistic model. The RMSE was 1.50 for this dataset. The forecast values were close to the actual values, indicating an impressive performance of the model used to estimate exchange rates.

Table 9. The in-sample forecast for the monthly values of the actual exchange rate data between the SGD and GBP with the forecasted values from Jan 2021 to August 2022.

Period (2021)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Original	1.81	1.84	1.86	1.85	1.87	1.87	1.87	1.87	1.85	1.85	1.83	1.81
Forecast	1.27	1.32	1.23	1.22	1.78	1.21	1.66	2.22	2.02	1.91	2.04	1.93
RMSE	0.12	0.12	0.14	0.14	0.02	0.15	0.05	0.08	0.04	0.01	0.05	0.03

Period (2022)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Original	1.83	1.82	1.79	1.77	1.72	1.7	1.67	1.66	---	---	---	---
Forecast	1.27	1.05	1.95	1.75	1.96	1.97	1.96	1.44	---	---	---	---
REMSE	0.13	0.17	0.04	0.01	0.05	0.06	0.07	0.05	---	---	---	---

Table 10 displays the result obtained for the forecast of monthly values of the actual exchange rate data between the THB and GBP with the forecasted via simulated log-logistic model. The RMSE was 12.07 for this dataset. One can observe that the forecast and the actual values are very close, establishing the adequacy of the log-logistic model in forecasting the exchange rate data.

Table 10. The in-sample forecast for the monthly values of the actual exchange rate data between the THB and GBP with the forecasted values from Jan 2021 to August 2022.

Period (2021)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Original	40.91	41.63	42.68	43.37	44.08	44.11	45.08	45.71	45.41	45.81	44.51	44.62
Simulated	45.96	47.57	49.97	41.43	42.13	47.27	40.68	44.53	48.77	45.75	44.40	46.34
RMSE	1.129	1.327	1.629	0.433	0.437	0.708	0.984	0.265	0.751	0.012	0.030	0.384
Period (2022)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Original	45.04	44.18	43.79	43.66	42.82	43.03	43.60	42.97	---	---	---	---
Simulated	47.99	46.19	46.87	41.35	45.21	44.50	45.23	41.03	---	---	---	---
RMSE	0.659	0.451	0.689	0.515	0.534	0.328	0.364	0.432	---	---	---	---

Accurate and efficient forecasting technique is a global challenge in finance portfolios. This is due to unprecedented volatility in financial data that is empirically challenging to forecast. A few forecasting techniques were developed to estimate the market assets to reduce the risks of trading in such an asset. However, these approaches have failed to produce accurate estimates relating to financial data, which necessitated the development of more forecasting techniques in the financial sector. The present research attempted to forecast the exchange rate data between the MYR, SGD, and THB against the GBP using probability distributions to assist investors and policymakers in reducing the high risks involved in trading in any of these currencies. The objective is to identify a more flexible probability distribution that can adequately fit and forecast the monthly exchange rate dataset from April 2005 to August 2022 with higher accuracy. The results showed that the log-logistic distribution provided good fit and better forecasts for all datasets under consideration among all competitive probability distributions.

The monthly exchange rate data between the MYR and GBP was subjected to ten probability distributions used in the finance literature to identify a better fit distribution. The result showed that the log-logistic model gives a better fit than other models based on statistical measures, such as the log-likelihood, AIC, BIC, K-S, C-V, and A-D. In addition, the simulation study was carried out to assess the performance of the log-logistic model. The simulations result confirmed the flexibility of the model. We used the model for the in-sample forecast for the MYR and GBP exchange rate from January 2021 to August 2022. The result revealed that the forecast values were very close to the actual values, with a root means squared error value of 1.66.

Similarly, the exchange rate data between the SGD and GBP was used to determine the best fit probability distributions among the competing models. The log-logistic distribution was found as the best-fitted distribution using statistical criteria, such as the log-likelihood, AIC, BIC, K-S, C-V, and A-D. The simulation results also verified the validity of the log-logistic distribution for the exchange rate data. The forecast values for the exchange rate data from this model were closely matched with the original data, with an RMSE value of 1.55.

In the same vein, the monthly exchange rate data between the THB and GBP was applied to the most common probability models in finance to find the most suitable fit model. The log-logistic distribution was chosen as the most suit model based on the results obtained from the goodness-of-fit tests among the candidate models. The simulation study agreed with the suitability of the model. Furthermore, the model provided good forecast values for the exchange rate data, with an error value of 12.07.

Similarly, researchers from Kenya developed a long-term exchange rate forecasting model for exchange rate data using quarterly data from 2007 to 2016. The new hybrid of quantile regression forest and Gaussian kernel was employed for statistical modeling. The developed probability model was used for in-sample forecasts from 2013 to 2016. The result with correct coverage was obtained based on a prediction interval of 95% and 30% [35]. Another study conducted in Ghana also applied probability distribution to forecast monthly exchange rate data from 2000 to 2017. The results showed that the lognormal distribution fitted the data more accurately than the competitive models [1]. In a study recently conducted in Nigeria, a new probability model was developed to estimate the yearly exchange rate data from 1995–2016. The model provides a better fit for the dataset [36].

This study investigated the practicability of probability distributions with financial application in the field of exchange rate fitting and forecasting. Most of the distributions performed well in describing the exchange rates under investigation. After testing several accuracy measures, we found that the log-logistic distribution provided the best fit among all competitive models. The accuracy of the model's forecasts was assessed using the root mean squared errors and revealed optimum errors. This meant that the minimum error of the forecasts obtained in each dataset confirmed that the model's propagated forecasted values were almost as accurate as the actual datasets. Hence, the distribution could be used in exchange rate forecasting to assist investors and all relevant stakeholders in predicting exchange rates for their decision-making.

4. Conclusion

In recent years, statistical models have become increasingly applied to analyze data in relevant areas, such as finance, economics, hydrology, education, medicine, and engineering. The probability distribution forecasting model for the exchange rate provides essential information in a stock market to assist investors and policymakers in decision-making. The most common statistical models in finance were applied to identify the distribution that can accurately forecast the exchange rate data of the three Southeast Asian countries' trading currencies against the British Pound Sterling. The data was divided into the training and the testing sets. All competing models were subjected to the training dataset to check which distribution the data follows. A variety of different accuracy metrics were used to choose a model that gives better fits to the datasets. The log-logistic distribution provided the best fit for all the exchange rate datasets among all competitive models in terms of goodness-of-fit. The Monte Carlo simulation tests were carried out to evaluate the performance of the fitted log-logistic distribution based on maximum likelihood estimates. Finally, we conducted in-sample forecasts for the three datasets to verify the practicability of the log-logistic model in forecasting the exchange rate data. The minimum error of the forecasts obtained in each dataset confirmed that the model's propagated forecasted values were almost as accurate as the actual datasets. The usefulness of this study can be applied to any other currency. In the future, we recommend that more extensions of the traditional probability distributions are needed to provide a more accurate forecast for the exchange rate data.

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